[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2257 **GC-3** **Your Roll No................**

Unique Paper Code : 32355101

Name of the Paper : GE – I Calculus

Name of the Course : **Generic Elective for Hons. Courses**

Semester : I

Duration : 3 Hours Maximum Marks : 75

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION I

1. Use $\varepsilon - \delta$ definition to show that

$$\lim_{x\to 3}(3x-7)=2.$$

2. Find the equations of the asymptotes for the curve

$$f(x)=\frac{x^3+1}{x^2}.$$

3. Find the Linearization of

$$f(x) = \sin x \quad \text{at} \quad x = \pi.$$

4. For $f(x) = x^3 - 3x + 3$

   (i) Identify where the extrema of 'f' occur.

(ii) Find where the graph is concave up and where it is concave down.

5. Use L'Hôpital's rule to find

$$\lim_{x \to \pi/2} \frac{1-\sin x}{1+\cos 2x}.$$

6. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

7. Find the length of the astroid

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi.$$

SECTION II

8. State Limit comparison test. Using the limit comparison test, discuss the convergence of

$$\int_1^{\infty} \frac{dx}{1+x^2}.$$

9. Identify the symmetries of the curve and then sketch the graph of

$$r = \sin 2\theta.$$

10. Solve the initial value problem for $\vec{r}$ as a vector function of t

Differential equation : $\dfrac{d^2\vec{r}}{dt^2} = 32\hat{k}$

Initial Conditions : $\vec{r}(0) = 100\hat{k}$

and : $\left(\dfrac{d\vec{r}}{dt}\right)_{t=0} = 8\hat{i} + 8\hat{j}$

11. Find the curvature for the helix

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k},\ a,b \geq 0\ \ a^2 + b^2 \neq 0$$

12. Write the acceleration vector $\vec{a} = a_T\hat{T} + a_N\hat{N}$ at the given value of t without finding $\hat{T}$ and $\hat{N}$ for the position vector given by

$$\vec{r}(t) = (-t\cos t)\hat{i} + (t\sin t)\hat{j} + t^2\hat{k},\quad t = 0$$

13. Show that $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

is continuous at every point except at the origin.

14. If $f(x,y) = \dfrac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$

(i) Find the domain of the given function f(x,y).

(ii) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

SECTION III

15. If $z = 5\tan^{-1} x$ and $x = e^u + \ln v$,

find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ using chain rule, when $u = \ln 2$, $v = 1$.

16. Find the directions in which the given function f increase and decrease most rapidly at the given point p_0. Then, find the derivative of the function in those directions.

$$f(x,y,z) = \frac{x}{y} - yz,\quad p_0(4,1,1)$$

17. Find parametric equations for the line tangent to the curve of intersection of the given surfaces at the given point.

$$\text{Surfaces : } x + y^2 + 2z = 4, \quad x = 1$$

$$\text{Point : } (1, 1, 1).$$

18. Find equations for the

(a) Tangent plane and

(b) Normal line at the point p_0 on the given surface

$$z^2 - 2x^2 - 2y^2 - 12 = 0; \quad p_0(1, -1, 4).$$

19. Find the absolute maxima and minima of the function $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0$, $y = 0$, $y + 2x = 2$ in the first quadrant.

20. If $f(x,y) = x\cos y + ye^x$, find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

21. If $f(x,y) = x - y$ and $g(x,y) = 3y$

Show that

(i) $\nabla(fg) = g\nabla f + f\nabla g$

(ii) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$